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# IMAGE EXPLOITATION USING WAVELETS STUDY

Westinghouse Electric Corporation

Frank Guillen, Dr. Martin G. Woolfson, Derek Roeum,  
and Herbert Ahmuty

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APPROVED:



MARK R. ROSIEK  
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13. ABSTRACT (Maximum 200 words)  This report details the results of a study that was conducted in the application of wavelet transforms to the processing of digital reconnaissance imagery. Although the major emphasis in the study centered on General Frame wavelets, orthonormal wavelets (Daubechies) were also implemented. The study results led to the implementation of a set of wavelet based tools to be included in the Image Processing Toolkit at Rome Laboratory. Some of the tools were designed to aid in the visualization of the interaction of the wavelet transform and imagery. Other tools were designed to provide different views of the imagery (such as pseudocoloration) and to enhance certain image features based on the localization property of the wavelet transform.			
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## 1.0 INTRODUCTION

A study titled "Imagery Exploitation Using Wavelets" was conducted at Westinghouse Electric Corporation to further the exploitation of digital reconnaissance imagery. The study results provide a set of wavelet based tools to be included in the Image Processing Toolkit. The tools were designed to aid in visualization of the interaction of the wavelet transform and imagery, to provide different views of the imagery (such as pseudocoloration) and to enhance certain image features based on the localization property of the wavelet transform. This Final Report provides some lessons learned during the development of the tools, a general description of the tools and recommendations for further work in this area.

## 2.0 OBJECTIVE

The objective of this study was to develop a set of image exploitation tools using wavelets. Earlier experimental work suggested that tailoring the mother wavelet to the task at hand could enhance the ability to extract certain image features. For example, this was shown in the extraction of road scarring in Synthetic Appature RADAR (SAR) imagery. In view of the above, two classes of wavelets were implemented in the study: general frame wavelets and orthonormal wavelets. The wavelets, which are covered in the next section, are applicable to most of the tools. Six tools were developed and are listed below:

- WAV\_SCA - Wavelet Scalogram
- WAV\_CEP - Wavelet Cepstrum (Originally named WAV\_CIC)
- WAV\_MAC - Wavelet Multiresolution Automatic Color Rendering
- WAV\_VOL - Wavelet Volumepass Filtering
- WAV\_TEN - Wavelet Transform Encoding
- WAV\_SOF - Wavelet Scale Orientation Filter

## 3.0 WAVELET FUNCTION AND IMPLEMENTATION

In a word, wavelets are locally supported functions labeled with scale and delay and comprising a basis for function space. First a mother wavelet (Figure 3.0-1) is selected according to an admissibility condition; then, the mother is labeled with scale and delay to generate the wavelets which comprise a basis (Figure 3.0-2).

Wavelets are attractive for the exploitation of digital reconnaissance imagery for many reasons. Wavelets may be used to discern the local frequency content of a signal. Data may be decomposed into its scale and spatial-orientation

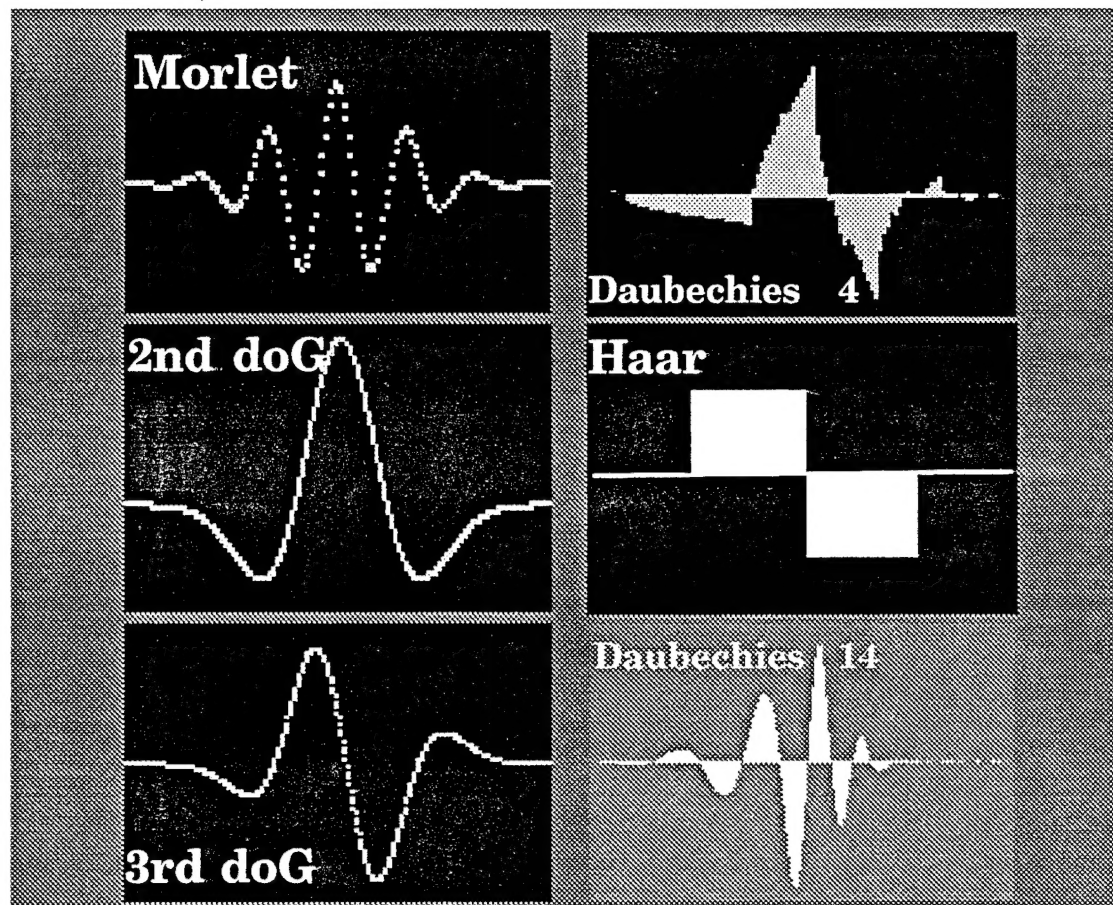


Figure 3.0-1 Example Mother Wavelets  $h(t)$

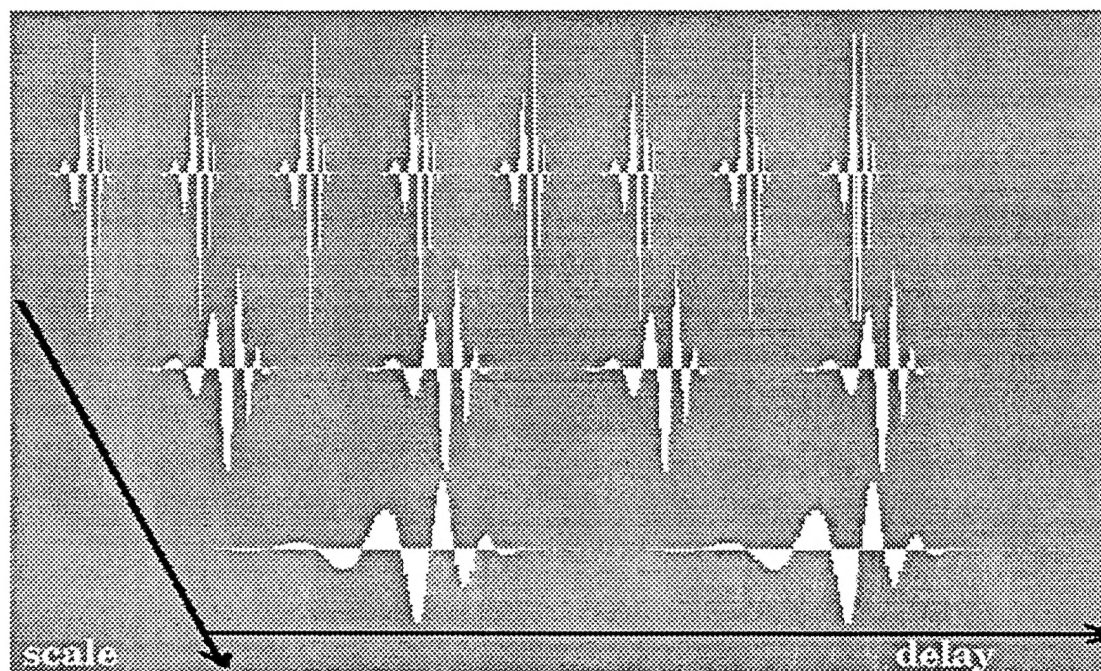


Figure 3.0-2 A Basis of Wavelets  $h_{\alpha,\tau}(t)$

components using wavelets [3]. The mathematical rigor of the wavelets admissibility and frame condition precludes the inadvertent loss of information or the introduction of artifacts. The frame condition [1] equips both general-frame and orthonormal wavelets with fast transforms for near-real time operation and efficient data storage.

By far the most important attribute of wavelets, however, is the latitude they offer to tailor the analysis basis to suit the application at hand. This is due to the rather lax admissibility condition on the mother wavelet [2] which requires that the inner product between the inverse magnitude of frequency and the magnitude squared of the Fourier transform of the mother be finite:

$$\int |\omega|^{-1} |\hat{h}(\omega)|^2 d\omega < \infty \quad \text{Eq. 3.0.1 Mother Admissibility}$$

This condition is satisfied by an infinite variety of functions. Figure 3.0-1 highlights the diversity among allowable mother wavelets. This freedom to tailor the wavelets to suit the application at hand makes it possible to invoke wavelet basis which, during transform, force application-important features to congeal in localized regions of influence in the wavelet domain. For image compression, coefficients outside the region of influence may be quantized to zero while, for image exploitation, coefficients in the region of influence may be filtered using volume-pass or cepstrum filters. However, in all of these applications, the common practice of tailoring the mother to cause qualitatively important data features to congeal in a localized region in the wavelet domain is exhibited.

An admissible mother  $h(t)$  is converted to a basis of wavelets by substituting  $t \rightarrow (t-\tau)/\alpha$  in  $h(t)$  and multiplying by  $1/\sqrt{\alpha}$  for normalization:

$$h_{\alpha,\tau}(t) = \frac{1}{\sqrt{\alpha}} h\left(\frac{t-\tau}{\alpha}\right) \quad \text{Eq. 3.0.2 Basis of Wavelets}$$

Scale  $\alpha$  being in the denominator in the argument provides the contractions and dilations of the mother while  $\tau$  being subtracted in the numerator in the argument provides time-shifted versions at each scale. A wavelet transform of an input function  $x(t)$  is accomplished by taking the inner product between the input function (or sequence)  $x(t)$  and all of the labeled wavelets and is a function of scale  $\alpha$  and delay  $\tau$ :

$$\text{Wavelet Transform } (x(t))[\alpha,\tau] = \langle x(t), h_{\alpha,\tau}(t) \rangle \quad \text{Eq. 3.0.3 Wavelet Transform}$$

While the mother wavelet must satisfy an admissibility condition, the labeled wavelets themselves must meet a frame condition [1], which governs the spacing in



the  $\alpha, \tau$ -grid, in order for the wavelet transform to be reversible and information-preserving. A generalized Parseval's Theorem, it requires a proportionality for each  $x(t)$  between the energy of a function in the wavelets domain and its norm in the input domain:

$$A \|x(t)\|^2 \leq \sum_{\alpha, \tau} |\langle x(t), h_{\alpha, \tau}(t) \rangle|^2 \leq B \|x(t)\|^2$$

$$A > 0, B < \infty \quad \text{Eq. 3.0.4 Frame Condition}$$

A fast wavelet transform (FWT) may be realized for both general-frame and orthonormal bases of wavelets by setting the  $\alpha, \tau$ -grid at the widest possible spacing (critical frame). This process is illustrated in Figure 3.0-3 where a square wave is forward - and then inverse - transformed for three  $\alpha, \tau$ -spacings.

To set a wavelet transform for maximum speed and efficiency, the  $\alpha, \tau$ -spacing (the dots on the left of Figure 3.0-3) is gradually "tightened" until the reconstruction (the function  $x(t)$  on the right of Figure 3.0-3) converges. The exact  $\alpha, \tau$ -grid spacing, which is as sparse as possible while maintaining reversibility, is called critical frame. All wavelet transforms (except those for scalograms) shall be set at critical frame.

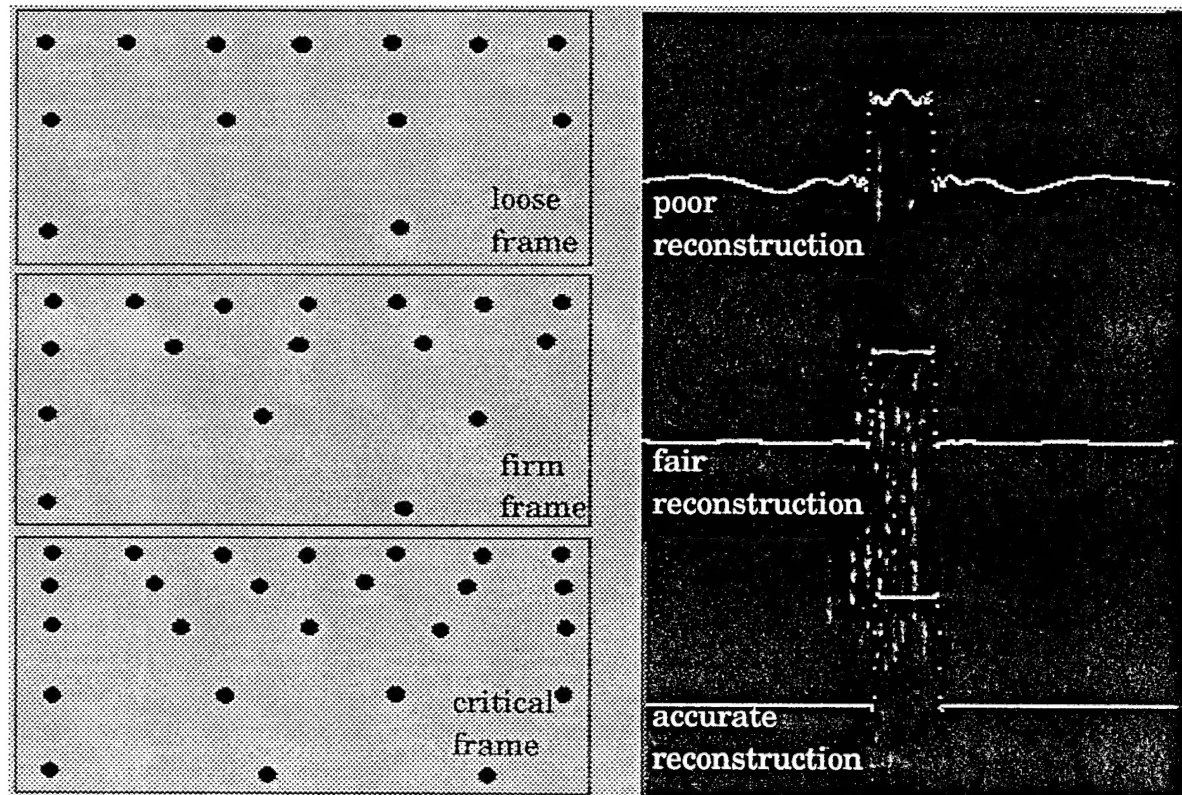


Figure 3.0-3 Setting a Wavelet Transform at Critical Frame.

### 3.1 General Frame Wavelets

The following General Frame Wavelets meet the frame and admissibility conditions and were implemented in this study :

Haar – The simplest of wavelets assuming the values of plus 1 and minus 1 over small consecutive intervals.

$$\text{Prototype: } h(t) = \begin{cases} 1, & 0 \leq t < \frac{1}{2} \\ -1, & \frac{1}{2} \leq t < 1 \\ 0, & \text{otherwise} \end{cases}$$



2SOPS – A derivative of the Haar wavelet using a second order low pass filter to define the wavelet shape.

$$\text{Prototype: } h(t) = \frac{t}{(t^2 + 1)^2}$$



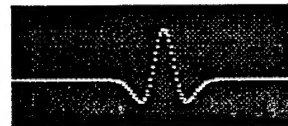
Morlet – Sine and cosine functions which are windowed by a Gaussian to limit their extent. The transform is typically implemented with complex numbers.

$$\text{Prototype: } h(t) = e^{-\frac{t^2}{2}} \cos(3.5t)$$



2nd Dog – Second derivative of a Gaussian.

$$\text{Prototype: } h(t) = (1 - t^2) e^{-\frac{t^2}{2}}$$



3rd Dog – Third derivative of a Gaussian.

$$\text{Prototype: } h(t) = (t^3 - 3t) e^{-\frac{t^2}{2}}$$



Additional developments on 2SOPS and the derivative of Gaussian Wavelets can be found in document "New Classes of Frame Wavelets for Applications"[4].

With a change in scale, the wavelet function approaches the input sample number in length. As a consequence, the number of operations involved in computing the inner products becomes very large and results in long processing

times for moderate size images. Two approaches were taken in order to speed up the process:

1. The first method employed the critical frame process discussed previously. For this method some of the mother wavelets were pre-computed at the various non-integer scale and delay values and used with table lookup procedures.
2. The second method performed the correlations using FFT methods as described below. The correlation between two function  $g$  and  $h$  is given by:

$$\text{Corr}(g,h) = \int g(t) * h(t + \tau) dt . \quad \text{Eq. 3.1.1}$$

The correlation value depends on the delay value  $\tau$ . For  $G(f)$ , the Fourier transform of  $g(t)$  and  $H(f)$ , the Fourier transform of  $h(t)$ , then

$$\text{Corr}(g,h) = F^{-1}(G(f) H^*(f)) \quad \text{Eq. 3.1.2}$$

where  $F^{-1}$  is the inverse Fourier transform and the asterisk refers to the complex conjugate of the transform value. In the implementation, the mother wavelets were pre-computed for various alpha (scale) values at zero delay. The resulting functions were transformed and stored as transformed values. The inner product values at each scale and delay were performed by computing the transform of the input function ( $g$ ) once and then performing the complex product operations for each alpha representation. The inverse transforms of the products produce the transform values as a function of both scale and delay.

As an example of the processing change, a 512 by 512 pixel image processed using the WAV\_MAC algorithm on a Silicon Graghics IP12 Processor (36MHz), required 59 hours processing time before implementation of the FFT method and 5.4 hours after implementation of the FFT method.

### 3.2 Orthonormal Wavelets

In addition to the general frame wavelets, the orthonormal 4 point Daubechies wavelet was also implemented and is used exclusively in the WAV\_TEN, WAV\_SOF, and WAV\_VOL tools and implemented with general frame wavelets in the WAV\_CEP tool. The pyramidal form of the transform and its inverse produce computation speeds that approach that of FFTs and permits fast inspection of the effects of the tool being used on the input image. Until such time that the general frame method can be sped up to yield comparable processing times, the orthonormal wavelets are the method of choice.

## 4.0 WAVELET TOOLS

### 4.1 WAV\_MAC

The purpose of this tool is to improve visualization by imparting a pseudo-coloration of the image. The colors are derived from the absolute value of the wavelet transform of the image partitioned into groups formed by consecutive scales. The lower consecutive scales are assigned red, the mid range consecutive scales are assigned green, and the high consecutive scales are assigned blue. The color weights at each pixel sum to one and are used as multipliers of the original image intensity values. Thus, features in the image are colorized by scale. With clipping, shadow areas remain black and bright targets are rendered as white.

WAV\_MAC utilizes the FFT method (described in the Wavelets Functions and Implementation section) to decrease processing time. With this method there are 8 (MAX\_ALPHA) scales produced in the wavelet domain. Scales 0-2 are assigned to RED, scales 3-5 are assigned GREEN, and scales 6-7 are assigned BLUE. These scale boundaries are set by the "#define" statements in the IPWavMAC() function source code and can be changed by the user for varying results. A block diagram and example of WAV\_MAC are given in Figure 4.1-1.

### 4.2 WAV\_TEN

The purpose of this tool is to demonstrate the potential for image compression using wavelets. The tool employs a two dimensional four point Daubechies wavelet routine and a histogramming technique based on the log of the absolute value of the wavelet transform data to choose a threshold level. A percentage of the data to be retained is input to the routine and this value is translated to a threshold value (transform data below this level is set to zero). The effects on the image are immediately visualized by reconstruction of the image with the thresholded transform data. The transform is output for storage to an external file so that subsequent processing can be done. The data are stored as floating point values. It is noted that even though the input data is 8 bits (size of a character, 1 byte), a larger dynamic range is required to represent the transformed data (size of a float, 4bytes). Studies would have to be conducted to determine the word size to represent the data. Further enhancements of this tool would be the incorporation of the compression algorithms operation on the sparse data set. A block diagram and example of WAV\_TEN are given in Figure 4.2-1.

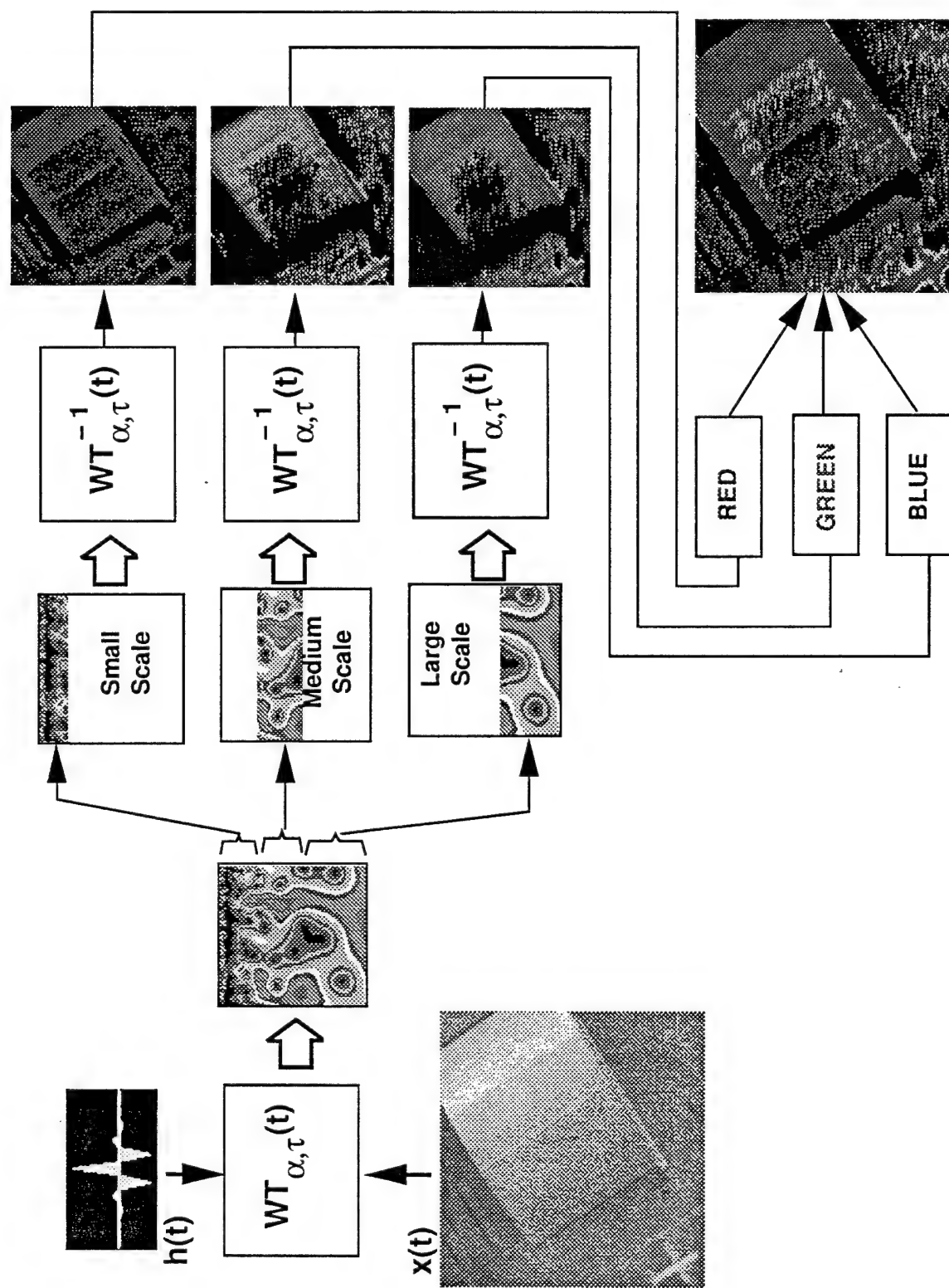


Figure 4.1-1 WAV\_MAC (Wavelet Multiresolution Automatic Color Rendering)

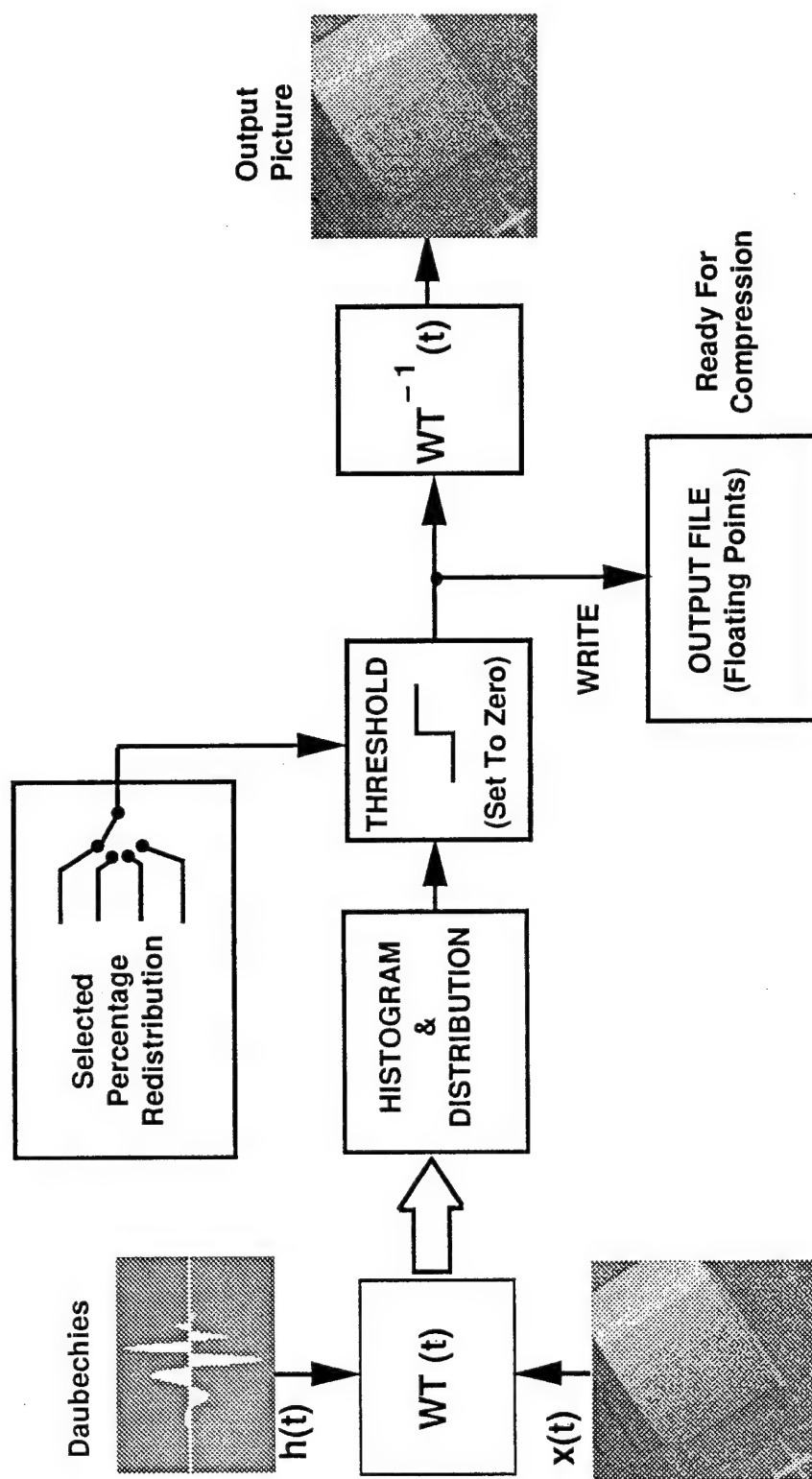


Figure 4.2-1 WAV\_TEN (Wavelet Transform Encoding)

### 4.3 WAV\_SOF

The Mallat decomposition provides a means for visualization of local horizontal, vertical or a combination of these data in an image. The process utilizes a four point Daubechies wavelet transform operating on the image and a transposed version of the image. The transform representation provides half of the total number of points (assumed a power of 2) to represent the smallest scale, half again for the next larger scale and so on. The image is processed with zero fill elements such that the picture fills a matrix to the nearest power of 2 in each dimension. For a matrix size  $2^m$  by  $2^n$ , the transform is presented in four quadrants of size  $2^{(m-1)}$  by  $2^{(n-1)}$ . By operation on the image and its transpose, one quadrant represents horizontal data, another vertical data, a third is a combination of horizontal and vertical, and the fourth a replication of the original image but reduced in resolution in each dimension by two. A replication occurs because the transform is orthonormal and applying the transform to the transpose of the transform results in an identity. The presentation shows the absolute value of the transform which is scaled to fill the 0-255 grey-scale range. Zero fill regions in the presentation (represented by black bands) could be eliminated in future versions of the IPWavSOF() function.

The WAV\_SOF program is designed to perform repeated Daubechies wavelet transforms on the reduced resolution replicate of the original image present in the transform output. The number of times the transform is repeated is controlled by argument "k\_max" of the function IPWavSOF(). Testing of the function indicate that "k\_max" > 3 gives limited to no added utility. The argument "k\_max" is presently hardcoded to 1 in the IPToolKit file "enhance\_handler.c". It is suggested that if future changes are made to the IPToolKit menus to allow multiple selections of "k\_max", the values be limited to 1 through 3. A block diagram and example of WAV\_SOF are given in Figure 4.3-1.

### 4.4 WAV\_CEP

The wavelet transforms offer a method of enhancing some image features on the basis of object size (or scale). Histograms of transform energy (the square of the transform values accumulated over delay for a particular scale) versus scale value exhibit a general fall off from large to small scale values not unlike Power Spectral Densities (PSD) derived in the Fourier domain. Logarithmic compression of the transform values (using the Cepstrum calculation) reduces the relative ratio of the large to small scale data. The resultant effect is enhancement of small features and edges.



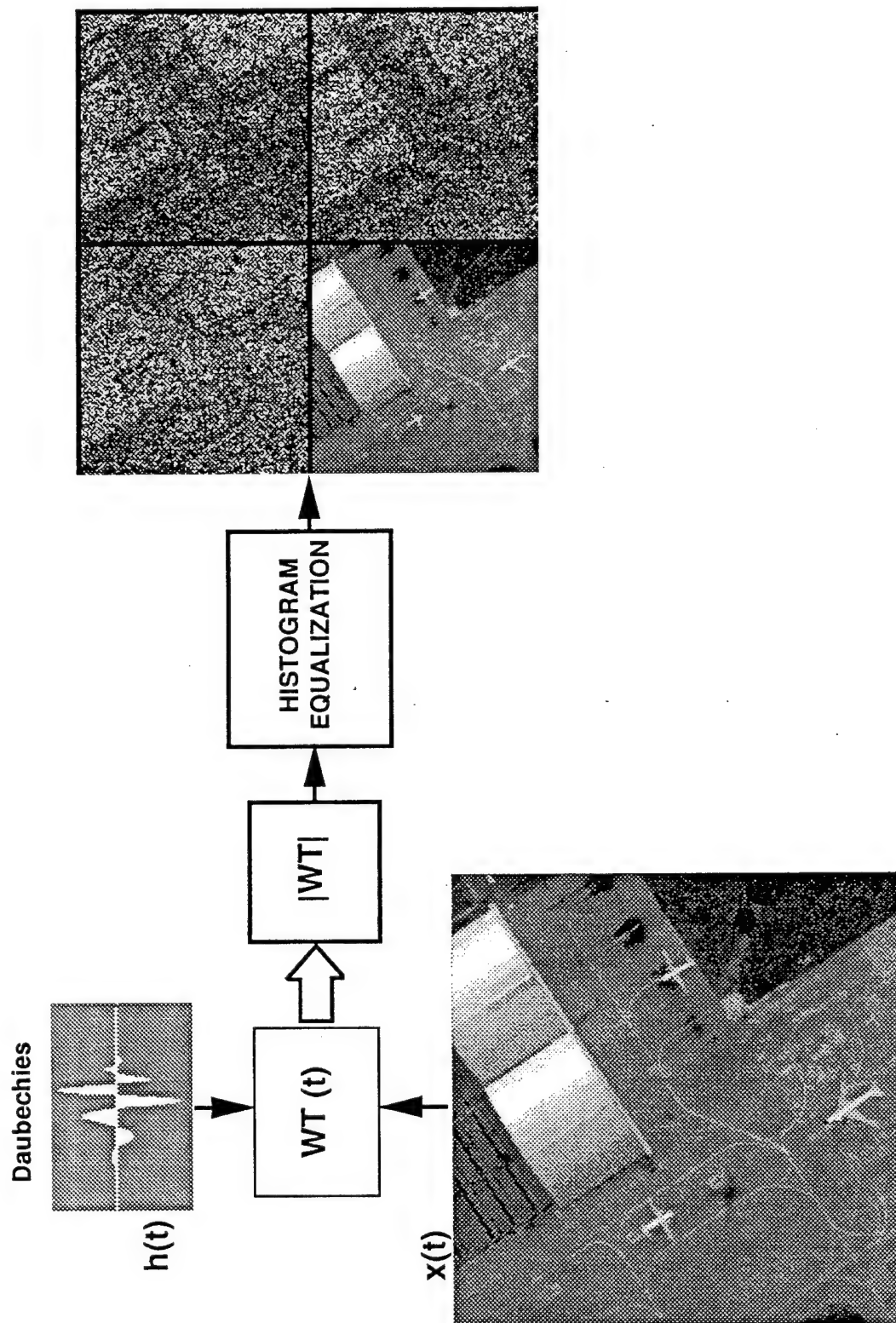


Figure 4.3-1 WAV\_SOF (Wavelet Scale-Orientation Filter)



The dyadic grid technique described earlier was implemented for the general frame wavelets of `IPWavCepstrum()`. Though this technique decreased execution time, the Morlet wavelet waveform could not be accurately implemented. Thus, the results for the Morlet wavelet show little utility. A block diagram and example of `WAV_CEP` are given in Figure 4.4-1.

#### 4.5 `WAV_SCA`

The wavelet transforms are labeled according to delay (the location of a local inner product calculation) and scale (the extent of the wavelet function for a compact wavelet and the region containing most of the wavelet function energy for the general form case).

A single image vector derived from a single row in the image is processed in this routine over a delay range corresponding to the vector length and over a range of scales. A plot of transform output at a particular scale versus delay produces a scalogram. Certain signal aspects such as bright point objects exhibit local high amplitude concentrations in the scalogram.

Because of the large absolute amplitude dynamic range that occurs with scale change, a non-linear mapping and color are required to maximize the visualization of the data. Incorporation of the required algorithms are reserved for a future update. The current process produces a quantized gray scale presentation using function `"sca_pix_hist"`. A block diagram and example of `WAV_SCA` are given in Figure 4.5-1.

#### 4.6 `WAV_VOL`

The wavelet transform is parameterized (labeled) according to amplitude, scale and delay. For some simple signal structures such as a range profile output of a High Range Resolution radar, the transform over some delay range (representing the profile length relative to a specific starting point), over some scales, and above some threshold level determine a volume in transform space. With a judicious choice of parameters, the inverse transform of the selected transform values exhibit a signature with less noise than the original and, possibly, somewhat less resolution. For more complex structures such as a 2D image, the delay is not as obvious and may be represented by rectangular regions (windows) to accommodate many objects of similar size dispersed over the image. In the

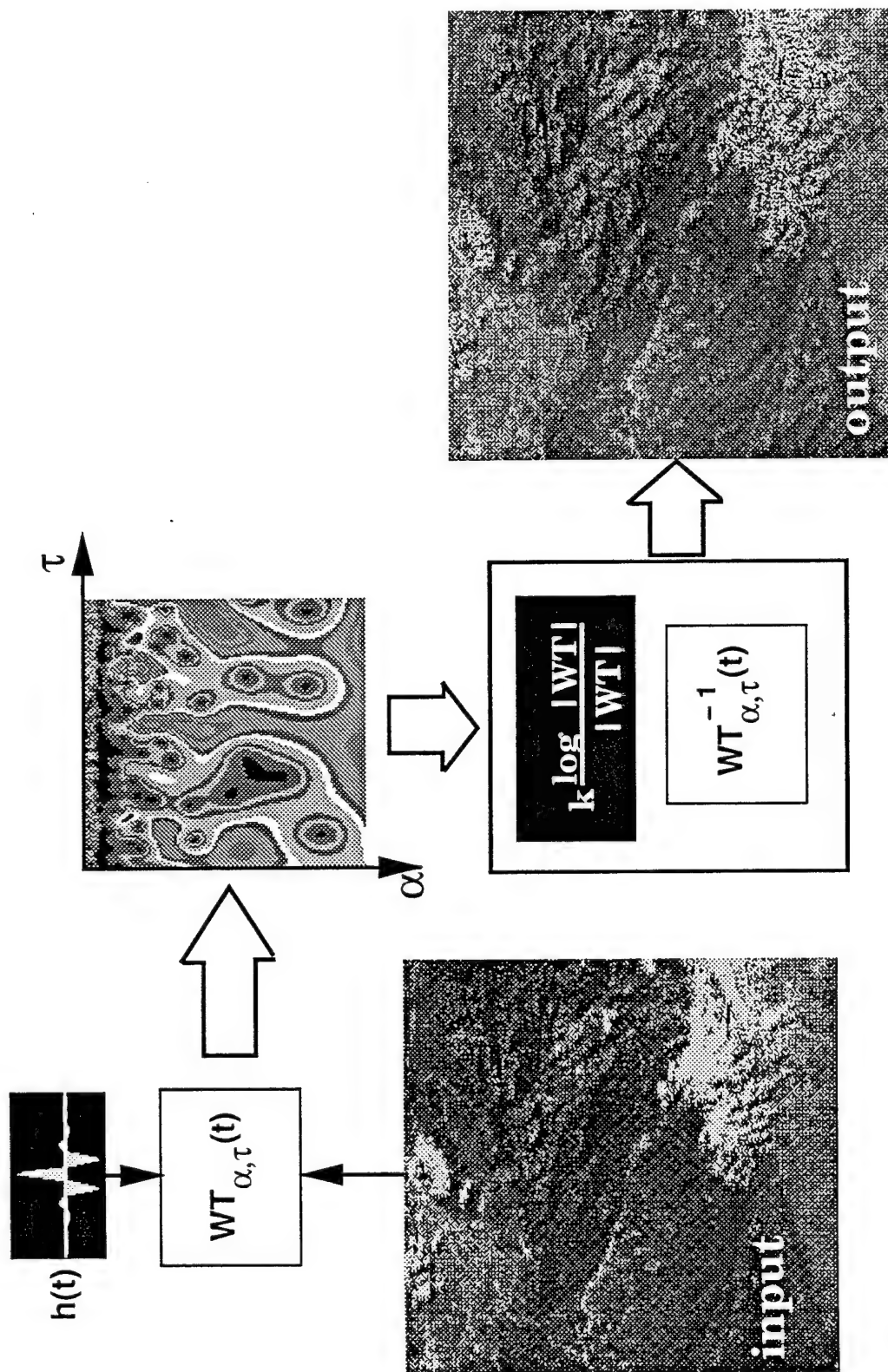


Figure 4.4-1 WAV\_CEP (Wavelet Cepstrum Filtering)

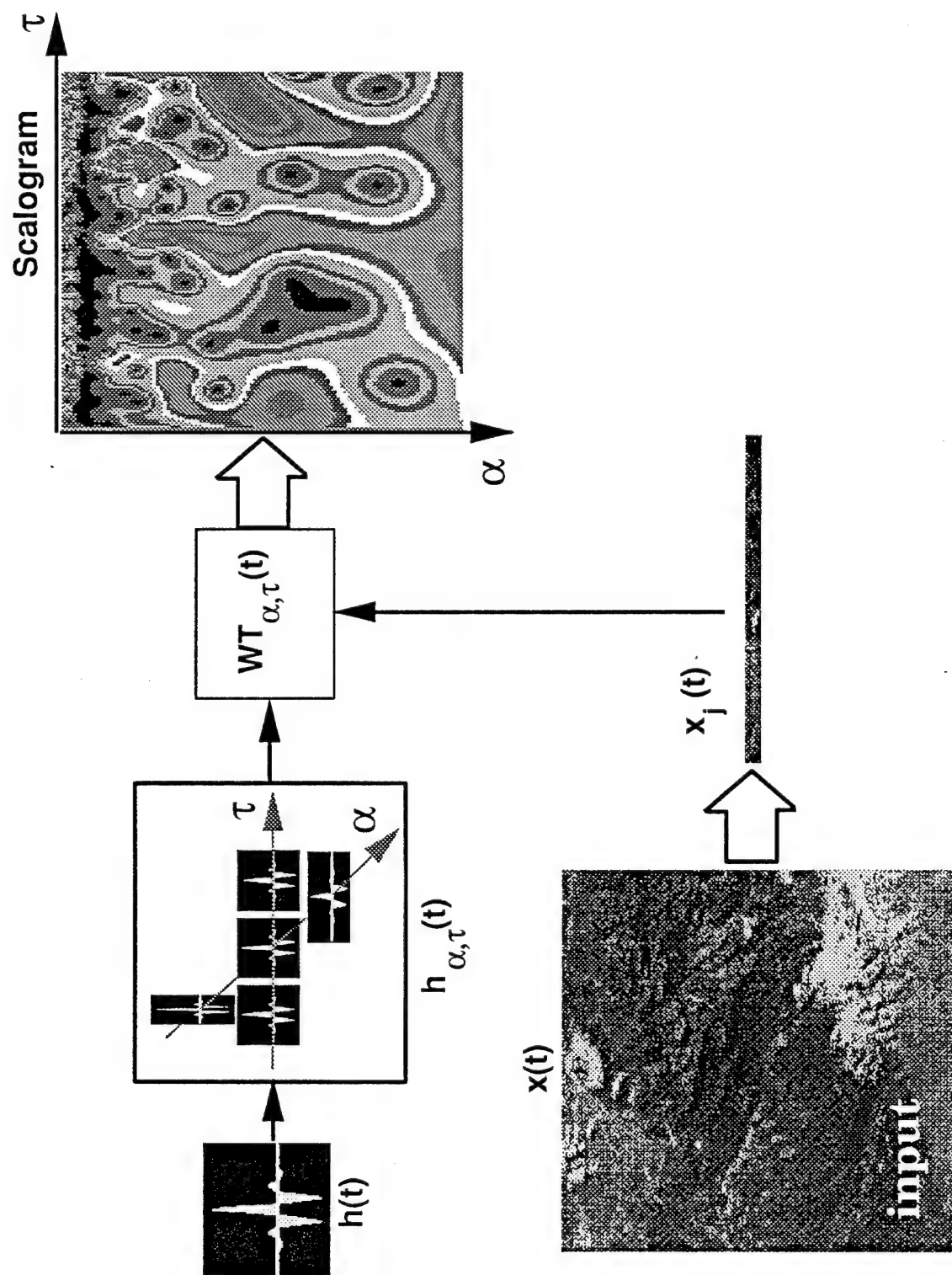


Figure 4.5-1 WAV\_SCA (Wavelet Scalogram)

current volume pass implementation, the selection of particular windows is reserved as a post-processing function (and is currently not implemented). The central focus of the processing is to manipulate the transform according to the scale data. Controls are provided to select scales in forms similar to that in the frequency domain: low pass (largest to moderate scales in consecutive order), high pass (moderate to smallest scale in consecutive order), band pass (a selected set with zero values for scale data not in the set) and stop band (a selected set of scales set to zero with others left intact). An additional control is provided so that the above selections can be added or subtracted from the total transform corresponding, for example, to boosting the data amplitude over some region in the frequency spectrum. These operations produce images with certain objects and features which are accentuated on the basis of scale (or object size) and amplitude. Further operations on the enhanced image could be provided by window filters as indicated above. A block diagram and example of WAV\_VOL are given in Figure 4.6-1.

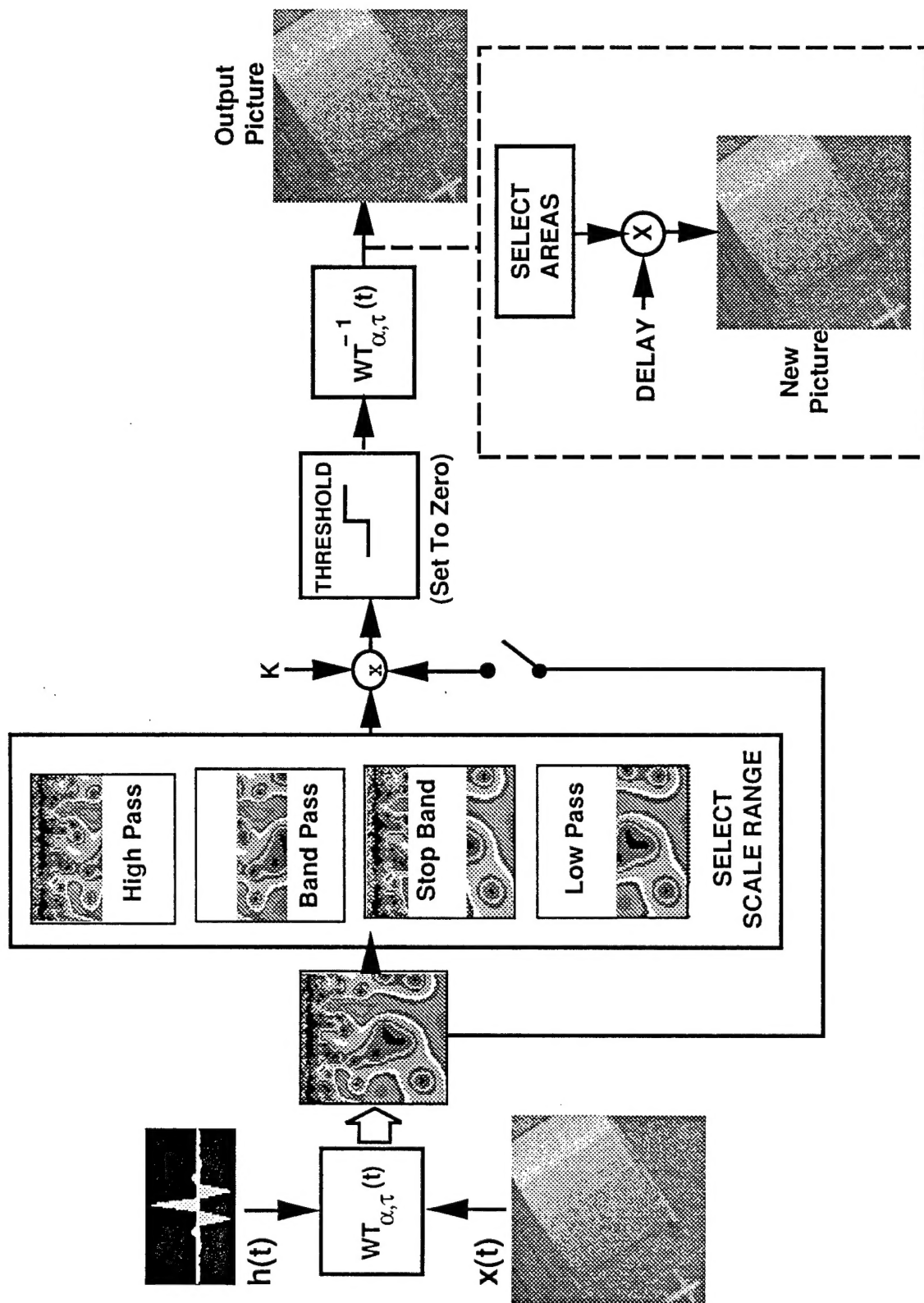


Figure 4.6-1 WAV\_VOL (Wavelet Volumepass Filtering)

## 5.0 SUMMARY

This report has presented background information and some of the tool specifics on the application of wavelet transform methods in the enhancement and visualization of digital reconnaissance imagery. The tools exploit the localization properties of the transform and the ability to manipulate data based on scale (or object size). The WAV\_TEN tool, for example, exploits the sparse nature of the discrete wavelet transform and the ability to represent the transform with a small subset of the computed values (the other values being set to zero). Other tools show the effect of the mother wavelet on the processed image by way of the presentation of specific image features.

## 6.0 RECOMMENDATIONS

Methods should be sought to improve the throughput rates for the wavelets which are currently being computed using general frame methods (and some adjuncts to these methods). Further study is required to determine the construction and approximation techniques for these waveforms that may be applicable.

Some minor cosmetic improvements in presentation of the data could be made on the IPWavSOF() function to account for image sizes which are not powers of two (the black regions resulting from zero fill of the original data can be eliminated by computation of their location in transform space). Similarly, non-linear intensity mapping and color would enhance the IPWavScalogram() function presentation.

The WAV\_MAC tool is hardcoded to divide the wavelet domain scales into 3 approximately equal groups (for red, green and blue). Future upgrade could include an interactive scroll bar to vary the scales into unequal groups.

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